## Wednesday 24 June 2015 - Morning <br> A2 GCE MATHEMATICS

## 4734/01 Probability \& Statistics 3

## QUESTION PAPER

## Candidates answer on the Printed Answer Book.

OCR supplied materials:
Duration: 1 hour 30 minutes

- Printed Answer Book 4734/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator


## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.


## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of $\mathbf{1 2}$ pages. The Question Paper consists of $\mathbf{4}$ pages. Any blank pages are indicated.


## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 A laminate consists of 4 layers of material $C$ and 3 layers of material $D$. The thickness of a layer of material $C$ has a normal distribution with mean 1 mm and standard deviation 0.1 mm , and the thickness of a layer of material $D$ has a normal distribution with mean 8 mm and standard deviation 0.2 mm . The layers are independent of one another.
(i) Find the mean and variance of the total thickness of the laminate.
(ii) What total thickness is exceeded by $1 \%$ of the laminates?

2 In a poll of people aged 18-21, 46 out of 200 randomly chosen university students agreed with a proposition. 51 out of 300 randomly chosen others who were not university students agreed with it. Test, at the $5 \%$ significance level, whether the proportion of university students who agree with the proposition differs from the proportion of those who are not university students.

3 A tutor gave an assessment to 6 randomly chosen new eleven-year-old students. After each student had received 30 hours of tuition, they were given a second assessment. The scores are shown in the table.

| Student | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st assessment | 124 | 121 | 111 | 113 | 118 | 119 |
| 2nd assessment | 127 | 119 | 114 | 110 | 120 | 122 |

(i) Show that, at the $5 \%$ significance level, there is insufficient evidence that students' scores are higher, on average, after tuition than before tuition. State a necessary assumption.
(ii) Disappointed by this result, the tutor looked again at the first assessment. She discovered that the first assessment was too easy, in fact being a test for ten-year-olds, not eleven-year-olds. She decided to reduce each score for the first assessment by a constant integer $k$. Find the least value of $k$ for which there is evidence at the $5 \%$ significance level that the students' scores have, on average, improved. [4]

4 A set of bathroom scales is known to operate with an error which is normally distributed. One morning a man weighs himself 4 times. The 4 values for his mass, in kg , which can be considered to be a random sample are as follows.

$$
\begin{array}{llll}
62.6 & 62.8 & 62.1 & 62.5
\end{array}
$$

(i) Find a $95 \%$ confidence interval for his mass. Give the end-points of the interval correct to 3 decimal places.
(ii) Based on these results, a $y \%$ confidence interval has width 0.482 . Find $y$.

5 Two guesthouses, the Albion and the Blighty, have 8 and 6 rooms respectively. The demand for rooms at the Albion has a Poisson distribution with mean 6.5 and the demand for rooms at the Blighty has an independent Poisson distribution with mean 5.5. The owners have agreed that if their guesthouse is full, they will re-direct guests to the other.
(i) Find the probability that, on any particular night, the two guesthouses together do not have enough rooms to meet demand.
(ii) The Albion charges $£ 60$ per room per night, and the Blighty $£ 80$. Find the probability, that on a particular night, the total income of the two guesthouses is exactly $£ 400$.
(iii) If $A$ is the number of rooms demanded at the Albion each night, and $B$ the number of rooms demanded at the Blighty each night, find the mean and variance of the variable $C=60 A+80 B$. State whether $C$ has a Poisson distribution, giving a reason for your answer.

6 In each of 38 randomly selected weeks of the English Premier Football League there were 10 matches. Table 1 summarises the number of home wins in 10 matches, $X$, and the corresponding number of weeks.

| Number of home wins | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of weeks | 0 | 1 | 2 | 8 | 8 | 9 | 7 | 1 | 2 | 0 | 0 |

Table 1

A researcher investigates whether $X$ can be modelled by the distribution $\mathrm{B}(10, p)$. He calculates the expected frequencies using a value of $p$ obtained from the sample mean.
(i) Show that $p=0.45$.

Table 2 shows the observed and expected number of weeks.

| Number of home <br> wins | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Totals |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Observed number <br> of weeks | 0 | 1 | 2 | 8 | 8 | 9 | 7 | 1 | 2 | 0 | 0 | 38 |
| Expected number <br> of weeks | 0.096 | 0.788 | 2.899 | 6.326 | 9.058 | 8.893 | 6.064 | 2.835 | 0.870 | 0.158 | 0.013 | 38 |

Table 2
(ii) Show how the value of 2.835 for 7 home wins is obtained.

The researcher carries out a test, at the $5 \%$ significance level, of whether the distribution $\mathrm{B}(10, p)$ fits the data.
(iii) Explain why it is necessary to combine classes.
(iv) Carry out the test.

## Question 7 begins on page 4.

7 A continuous random variable $X$ has probability density function

$$
\mathrm{f}(x)=\left\{\begin{array}{cl}
k x & 0 \leqslant x<2, \\
\frac{k(4-x)^{2}}{2} & 2 \leqslant x \leqslant 4, \\
0 & \text { otherwise },
\end{array}\right.
$$

where $k$ is a constant.
(i) Show that $k=\frac{3}{10}$.
(ii) Find $\mathrm{E}(X)$.
(iii) Find the cumulative distribution function of $X$.
(iv) Find the upper quartile of $X$, correct to 3 significant figures.

## END OF QUESTION PAPER

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